Realization of Daubechies transform using lattice structure

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Plan of presentation

1. Introduction
   - Problem statement
   - Goal of research
2. Theory
   - Lattice structure
   - Neural realization of wavelet transform
   - Teaching the network
3. Experimental validation
   - Testing method
   - Results
4. Conclusion

Problem statement

Wavelet transforms play an important role in processing, compression and analysis of signals. Wavelet transform is linear, similarly to DFT, DCT, DST and DHT. However it uses different basis functions. There is no universal basis function for a wavelet transform. It is important, that chosen wavelet closely corresponds to characteristics of analysed signal. It is possible to synthesize best wavelet function suitable for particular task using adaptive methods. Neural networks offer such possibility. Especially promising are the fast multilayer linear neural networks that are able to realize wide class of linear transforms.

Goal of research

Main goal was practical verification of neural approach to wavelet synthesis, by showing that neural network is able to learn existing Daubechies wavelet transforms. Network based on a lattice structure was chosen. It is a preliminary step for further research.

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Basic element of lattice structure

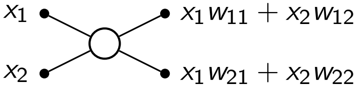
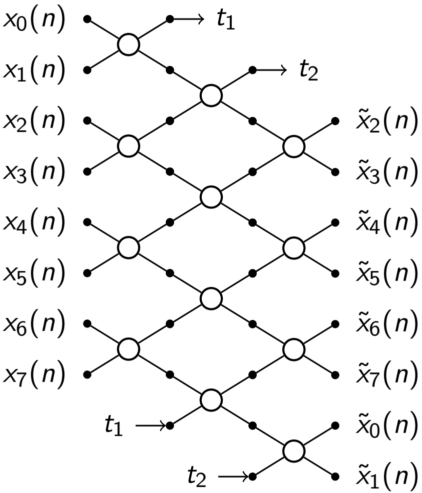


Figure: Basic structural element of lattice structure

Operation performed by this element can be treated as a 2-by-2 matrix multiplication:

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = P \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \text{, where } P = \begin{bmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \end{bmatrix} . \quad (1)$$

Lattice structure for realization of a wavelet transform



Neural realization of wavelet transform

Neural network was constructed based on the lattice structure. Most important properties :

- Weights of all neurons within one layer are identical.
- Neurons on the edge of a layer are wrapped around
- Neurons in the network are sparsely connected

Network properties

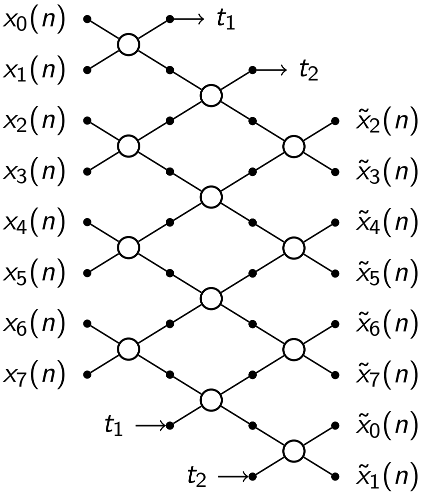
Network's ability to perform particular wavelet transform depends on number of layers. To calculate m-tap transform, m / 2 layers are necessary. Implementation of the network allows to operate on any signal of even length. It is possible thanks to identical weights of neurons in one layer.

Realization of inverse transform

Same network can be used for calculating the inverse transform. To achieve this, network direction should be reversed (inputs become outputs) and weight matrices, denoted in Equation 1 as P, should be replaced with inverse matrices:

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = P^{-1} \cdot \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \text{, where } P = \begin{bmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \end{bmatrix} . \quad (2)$$

Lattice structure for realization of a wavelet transform



Learning method

Supervised teaching was used to teach the network – for each input signal expected output was known. Error function minimized during learning process is:

$$E = \sum_{i=0}^{k-1} \frac{(y_i - d_i)^2}{k} \quad , \tag{3}$$

where y_i denotes signal value on i-th output of network, d_i denotes expected value for that output and k denotes number of outputs.

Learning algorithm

Network was taught using backpropagation algorithm. Weight modification was performed according to steepest descent algorithm:

$$w_{n+1} = w_n - \eta \nabla E(w) \quad , \tag{4}$$

where w_n is weights vector in n-th iteration, η is the learning step and $\nabla E(w)$ is error function gradient calculated in respect to network weights.

Expected results

Knowledge of network weights after completed learning process allows to calculate coefficients of linear filter realized by network. It will be demonstrated, that during teaching process weights converge to such values, that they reconstruct existing Daubechies filters.

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Testing method

Existing wavelet transforms – Daubechies 4, 6 and 8 - were chosen for experiments.

Properties of training and testing sets:

- Each signal in a set is a 16-element vector with each component chosen randomly from range [-1, 1]
- Each set consists of 1000 randomly generated signals and their transforms
- Training and testing sets are different

Learning process was terminated after reaching error less than10^{-10}.

Results of learning Daubechies 4 transform

	Filter coefficients			
	h_0	h_1	h_2	h_3
expected	0.4829	0.8365	0.2241	-0.1294
learned	0.4829	0.8365	0.2241	-0.1294
error [$\cdot 10^{-4}$]	0.0550	0.0497	-0.1258	0.0609

Table: Daubechies 4 low-pass filter coefficients

Results of learning Daubechies 6 transform

	Filter coefficients		
	h_0	h_1	h_2
expected	0.3327	0.8069	0.4599
learned	0.3326	0.8069	0.4598
error [$\cdot 10^{-4}$]	-0.1215	-0.1248	-0.0483
	h_3	h_4	h_5
expected	-0.1350	-0.0854	0.0352
learned	-0.1350	-0.0854	0.0352
error [$\cdot 10^{-4}$]	0.0423	0.0994	-0.0602

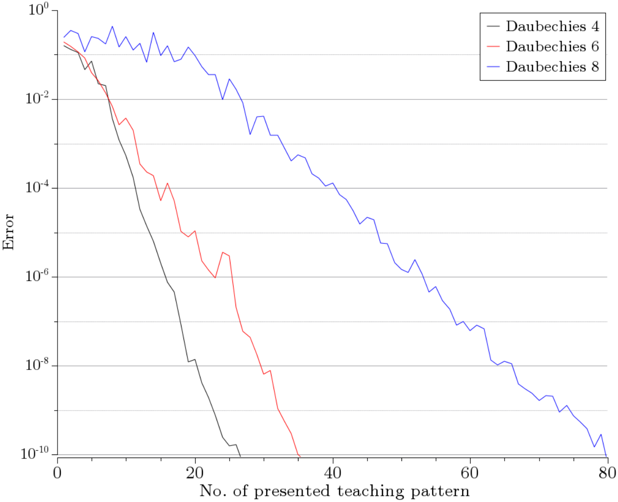
Table: Daubechies 6 low-pass filter coefficients

Results of learning Daubechies 8 transform

	Filter coefficients			
	h_0	h_1	h_2	h_3
expected	0.2304	0.7148	0.6309	-0.0280
learned	0.2303	0.7148	0.6308	-0.0279
error [$\cdot 10^{-4}$]	0.1091	-0.0183	0.1347	0.0078
	h_4	h_5	h_6	h_7
expected	-0.1870	0.0308	0.0329	-0.0106
learned	-0.1870	0.0308	0.0328	-0.0106
error [$\cdot 10^{-4}$]	0.0864	-0.0038	-0.0776	0.0335

Table: Daubechies 8 low-pass filter coefficients

Learning speed



General results

Transform	Presented patterns	Error	
		minimal	average
Daubechies 4	27	$1.2488 \cdot 10^{-11}$	$6.9721 \cdot 10^{-11}$
Daubechies 6	36	$2.0228 \cdot 10^{-11}$	$1.4539 \cdot 10^{-10}$
Daubechies 8	80	$3.6894 \cdot 10^{-11}$	$1.7562 \cdot 10^{-10}$

Table: Learning time and results obtained on testing set

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Conclusion

Experimental validation proved the ability of proposed neural network to learn selected wavelet transforms belonging to Daubechies family. Learning speed shows convergence of learning process in small number of iterations.

Obtained results are promising and allow to suspect, that proposed structure possesses potential ability to synthesize new orthogonal wavelet transforms.

End

Questions?